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6. Author(s). Robert W. Scarstein* and Anthony M. J. Davis*				7. Performing Organization Name(s) and Address(es). *The University of Alabama Tuscaloosa, AL 35487-0286			
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PENETRABLE WEDGE ANALYSIS
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ROBERT W. SCHARSTEIN, PRINCIPAL INVESTIGATOR
DEPARTMENT OF ELECTRICAL ENGINEERING

AND

ANTHONY M. J. DAVIS, CO-PRINCIPAL INVESTIGATOR
DEPARTMENT OF MATHEMATICS

The University of Alabama
Tuscaloosa, Alabama 35487-0286
205-348-1761

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The continuation of this wedge scattering research is proceeding as proposed, with several significant accomplishments and findings to date. The nature of this rigorous mathematical approach to scattering problems is such that original formulation is very important and usually very slow to evolve. For example, the previous work summarized in the report [61] had several abandoned and potentially useful starts, while the finished product is substantial.

I. JOURNAL PAPER

A journal paper titled "Mellin Transform Solution for the Static Line-Source Excitation of a Dielectric Wedge" has been submitted, revised, and accepted for publication in the *IEEE Transactions on Antennas and Propagation* [62]. A conference paper also summarizing these results was presented in July 1993 at the *1993 Radio Science Meeting* in Ann Arbor, MI.

II. GENERAL PENETRABLE WEDGE

In order to make the most significant progress possible, our initial philosophy during this first quarter of this research has been to concentrate on the most difficult aspect - the truly arbitrary, penetrable wedge. The solution to the impedance boundary wedge could then be extracted from the more general (and valuable) result. I have had scientific dialog with two leading scattering theorists, Egon Marx [20-22,28] and Ismo Lindell [63], who

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have made recent contributions to this and related wedge scattering problems. I would summarize the current (August 1993) thinking on the problem to be:

- (1) The penetrable wedge problem remains one of the great unsolved canonical scattering geometries.
- (2) Numerical "solutions" are still proliferating, but are too messy and removed from the physics to offer any important insight into the wave mechanisms that we hope to uncover and understand.

Based on last year's work [61] with the Kontorovich-Lebedev transform, which is a superposition of cylindrical waves, a general superposition of arbitrary plane waves has been investigated as a suitable field representation. Such a (spatial) spectrum of propagating ($|k_x| < k_j$) and evanescent ($|k_x| > k_j$) plane waves

$$\psi_j(x, y) = \int_{-\infty}^{\infty} dk_x \Psi_j(k_x, k_y) e^{i(k_x x + k_y y)}$$

where $k_x^2 + k_y^2 = k_j^2$ and $j = 1, 2$ denotes the two *different* media, may work in principle. A rather novel approach is to apply a conformal transformation to the homogeneous wedge region to obtain an effectively inhomogeneous half-space region. This results in a spatially varying wavenumber appearing in the transformed Helmholtz equation

$$(\nabla^2 + \kappa^2(\vec{r})) \psi(\vec{r}) = 0.$$

This approach demonstrates all too well a conservation of work principle: the problem can be cast in terms of the nice (familiar) Helmholtz operator with troublesome wedge boundaries or as a simple planar boundary that constrains a messy partial differential operator.

The lack of a rigorous solution technique to apply to the truly arbitrary wedge is disappointing to us, and is probably *a posteriori* expected by some colleagues. Fortunately, we can now confidently proceed with a new appreciation and respect for our primary goal of this year's research project – the mathematical solution of the impedance boundary wedge.

III. PHYSICAL IMPEDANCE BOUNDARY CONDITION

The coupled difference equations (14), (17), and (18) on page 27 of the previous report [61] are solved by first eliminating the function $C(\nu)$:

$$A(\nu) \sin(\nu\pi) + B(\nu) \cos(\nu\pi) = \cos[\nu(\phi' - \pi)]$$

$$\begin{aligned} \frac{k\eta}{2} \{ & A(\nu - 1) \sin[(\nu - 1)\alpha] + B(\nu - 1) \cos[(\nu - 1)\alpha] \\ & + A(\nu + 1) \sin[(\nu + 1)\alpha] + B(\nu + 1) \cos[(\nu + 1)\alpha] \} \\ & - A(\nu) \cos(\nu\alpha) + B(\nu) \sin(\nu\alpha) = 0. \end{aligned}$$

Application of an integral operator, such as the Laplace or Mellin inversion formula, eliminates the troublesome $\nu - 1$ and $\nu + 1$ and yields purely algebraic equations in the coefficient functions $A(\nu)$ and $B(\nu)$. Finding such a transformation that also results in explicit analytic expressions that can be conveniently carried through the inverse Kontorovich-Lebedev transform is an arduous task. Our present status is that of completed inversions via both the Laplace and the Mellin formulae, with results to date partially encouraging but still too complicated. We have identified several alternatives, including an asymptotic approach that lets $\nu = i\tau$ as it appears in the Kontorovich-Lebedev inversion, and concentrating on large τ . Adding to our motivation is the comforting knowledge that our well-defined boundary value problem possesses a unique solution which is defined in a single, homogeneous region of space. Also, the formulation and execution of the mathematics is rigorous and complete.

IV. INHOMOGENEOUS OR PSEUDO-IMPEDANCE BOUNDARY

The source of the shifting of ν to $\nu - 1$ and $\nu + 1$ in the above case of the Leontovich boundary condition

$$\psi(r, \alpha) + \frac{\eta}{r} \frac{\partial}{\partial \phi} \psi(r, \alpha) = 0$$

is the $1/r$ factor from the normal derivative

$$\frac{\partial}{\partial n} = \frac{1}{r} \frac{\partial}{\partial \phi}.$$

The observation on page 28 of [61] that an inhomogeneous surface impedance that is proportional to radius

$$\eta = r\eta'$$

avoids any difference equation in the transform coefficient functions. In this special case, much asymptotic progress and physical interpretation can be accomplished, as indicated in [61]. I did not emphasize this particular avenue, since it *appears* to be a problem contrived to fit a solution, rather than the solution to a real physical problem. Although Felsen and Marcuvitz [65] clearly recognized the analytic attractiveness of this "pseudo-impedance" wedge circa 1973, I only recently (today!) discovered their passing mention of it. They make no apology at all for any detachment from "reality." Perhaps I should not be so quick to dismiss this curious variation on the impedance boundary wedge. In any event, we can always return to this easier problem at a later date.

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